Postprint of: J.K. Vanclay, 1988. A stand growth model for cypress pine. *In*: J.W. Leech, R.E. McMurtrie, P.W. West, R.D. Spencer and B.M. Spencer (eds) *Modelling Trees, Stands and Forests*. School of Forestry, Univ. Melbourne, Bulletin No. 5, p. 310-332.

A Stand Growth Model for Cypress Pine

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Summary

A deterministic growth model for uneven-aged monospecific stands of cypress pine is presented. It is implemented as a cohort model and comprises equations to (1) predict stand basal area increment, (2) distribute stand increment among component trees, (3) estimate potential diameter increment to check for excessive distributed increments, (4) predict mortality, and (5) predict regeneration.

Introduction

White Cypress Pine (*Callitris glaucophylla* Thompson Johnson syn *C. glauca* R. Br. ex R. T. Baker & H.G. Sm.) commonly occurs as monospecific stands throughout southern inland Queensland. A growth model for these stands is being developed, but is currently incomplete and is not yet validated. This paper discusses the present status of the model.

Requirements of the model

The model is principally required for yield regulation applications, but

1. must be equally useful for investigative, yield prediction, and forest management planning purposes;
2. must be equally applicable to uneven-aged and even-aged (age unknown) stands; and
3. must be sufficiently flexible to enable it to utilize data derived from temporary or permanent plots which may be rectangular or circular and of fixed or variable area.

Data

During 1937-40 and 1955-58, a total of 117 permanent plots were established on three State Forests, using a systematic scheme with random origin. The plots are c. 0.4 hectare (1 acre) rectangular plots subdivided into four quadrats on which each tree taller than c. 3 metres (10 feet) is individually numbered, tagged and measured. Plot measures record the d.b.h. of every stem at every measure, but heights and other parameters are recorded less frequently. Heights of stems smaller than three metres are recorded for each of ten c. 10 square metre (2.5 milliacre) regeneration plots located along the centreline of each plot. Measurements were initially carried out every 2-3 years,

but the current prescription is to measure every six years. Additional measures are carried out at time of logging or silvicultural treatment.

Additional data from thinning experiments are available, but are unsuited to model development as the subjective location of these plots may introduce bias.

The 117 permanent plots were grouped according to site quality and logging history, and a stratified random sample of ten percent of the plots was reserved for validation. These reserved plots were not used in any way during the development of the model.

Modelling Approach

The requirements dictate that the model be deterministic, and suggest three possible approaches:

Stand Table Projection

This approach represents the stand as a stand table. Each year increment causes a certain proportion of the stems in any class to move up into the next class; mortality removes some of the stems from each class; and recruitment inserts some stems into the smallest class. The method has two inherent problems:

1. When the stand table is 'first compiled from inventory data, and actual diameter of each stem is lost in the class mean. This may represent the gain or loss of several years of increment, a serious loss of precision.
2. When a stem is recruited into any class, the stem immediately assumes the diameter of the centre of the class, which may introduce positive bias. These problems may be partially overcome by reducing the class width or by fitting spline curves to the stand table.

Distance Independent Individual Tree

This approach represents each tree as an entity, but takes no account of its placement in the stand (Munro 1974). It can be very effective for simulating the growth of stems on small plots, but exhibits two disadvantages:

1. the approach is not amenable to the use of data derived from prism sampling, and
2. the approach cannot model low levels of mortality unless a stochastic approach is adopted.

Cohorts (Reed and Clark 1979, Vanclay 1983, p.64)

These are groups containing an arbitrary number f supposedly identical trees. This approach bears some resemblance to stand table projection, except that

1. the cohort boundaries are not fixed, but are determined by the distribution of the trees in the stand,
2. the true mean diameter of the cohort is used, rather than the class mean, and
3. trees never move to another cohort, as the class boundaries rather than the trees move as the stand grows.

Examples of cohort models include those of Clutter and Allison (1974) who used 25 cohorts each with equal frequency, and or Alder (1977) who used deciles. These approaches are efficient for even-aged stands, particularly where mortality is negligible. However, in uneven-aged stands, the preponderance of small stems demands a different approach to forming cohorts. Leary (1979) presents a 'relativistic' approach which is used in the USDA (1979) model, but Vanclay (1983, p.73) argues that it is more efficient to group stems according to criteria analogous to the multiple range test.

The cohort approach is most suited to the present application. Thus the Cypress Pine Stand Growth Model is based on the cohort approach, and admits a maximum of 150 cohorts. When commencing a simulation, the model places trees in an existing cohort if its diameter is within 2 mm of the cohort mean, otherwise a new cohort is initiated. Generally, trees remain within the cohort for the duration of the simulation. However, to reproduce the heteroscedastic nature of increment, large increments or large numbers of stems in any cohort may cause it to split into two new cohorts, one of faster and one of slower growers. Conversely, cohorts may be merged when the simulation gives rise to two or more cohorts with trees of the same size. New cohorts are also initiated regularly to account for regeneration and recruitment. This approach is illustrated in Figure 1.

Functional Relationships

Three components of growth and change are addressed: survivor increment, mortality and regeneration.

Increment

A variety of approaches are available for calculation of increment (Vanclay 1983, p.85 et seq.):

1. Increment can be calculated at the tree level (as individual tree increments *which* are summed to obtain stand increment), or at the stand level (stand increment is calculated and shared among the component stems).
2. Increment can be calculated as diameter increment or basal area increment.
3. Increment can be predicted directly as a growth equation, or indirectly as a yield equation.

Vanclay (1983, p. 85-90) argues that the choice of diameter or basal area increment, and of a growth or yield equation, does not influence the accuracy of the model, and that this decision may be based on the utility of the resulting equations.

The choice between tree growth prediction and stand growth prediction is more difficult. If tree growth is predicted, it is necessary to employ some competition index and ensure a logical growth response at the stand level. Literally hundreds of competition indices have been published (Vanclay 1983, p. 102 et seq.), but most are no better than stand basal area (Johnson 1973, Martin and Ek 1984).

The alternative is to predict stand increment and share this among the component trees. This approach is a particularly robust

technique, and is effective for monospecific stands. However, determining the distribution of stand increment among individual trees may be complex.

Inventory Data: Horizontal Point Sample (BAF= 2) Dbhob (cm)	Year 0 Cohort List		Year 1 Cohort List		Foot Note
	No/ha	Dbhob (cm)	No/ha	Dbhob (cm)	
42.3	8.54	42.3	8.54	42.5	1
24.0	26.53	24.0	26.53	24.3	
19.8	38.97	19.8	38.97	20.1	
14.1	76.85	14.1	76.85	14.4	2
11.5	115.53	11.5	57.51	11.9	3
			57.51	11.8	
6.3	384.96	6.3	189.59	6.8	
			189.59	6.6	
			195.00	2.0	4
0.005 ha Plot Ht (m)	No/ha	Ht (m)	No/ha	Ht (m)	
2.5	200.00	2.5			4
1.3	200.00	1.3	195.00	1.6	5
0.6	200.00	0.6	195.00	0.7	
			5.10	0.1	6

Footnotes.
1. Diameter increment, negligible mortality.
2. Diameter increment and mortality.
3. Cohorts split if stocking or increment exceed user's specifications (here 100 stems/ha).
4. Regeneration reaching 3.0 m height is recruited to main model at 2.0 cm dbhob.
5. Height growth and mortality in regeneration.
6. New regeneration appearing.

Figure 1 Example of Cohort Model.

Stand Increment

The stand level approach is employed in the cypress pine stand growth model. As mortality and recruitment are predicted independently, the model requires the stand increment to be predicted as accretion, the increment on both surviving and dying trees but excluding recruitment. Accretion should decrease as basal area diverges from an optimum and increase as site quality improves. Few equations describing stand increment have been published, and most researchers employ a simple quadratic or the Chapman-Richards equation (Chapman 1961, Richards 1959). A new equation is proposed (Fig. 2):

$$SBAI=0.04637BA^{1.094}*\exp(0.0074028ASQ-0.2258BA)*\exp(0.4072TST^{2}e^{-TST})$$

where SBAI is stand basal area accretion ($m^2ha^{-1}y^{-1}$), BA is stand basal area (m^2ha^{-1}), SQ is site form[1] (m) and TST is time since last silvicultural treatment (years).

[1] Site form is the expected height of a 25 cm dbhob tree, and is analogous to site index. (Vanclay and Henry 1988).

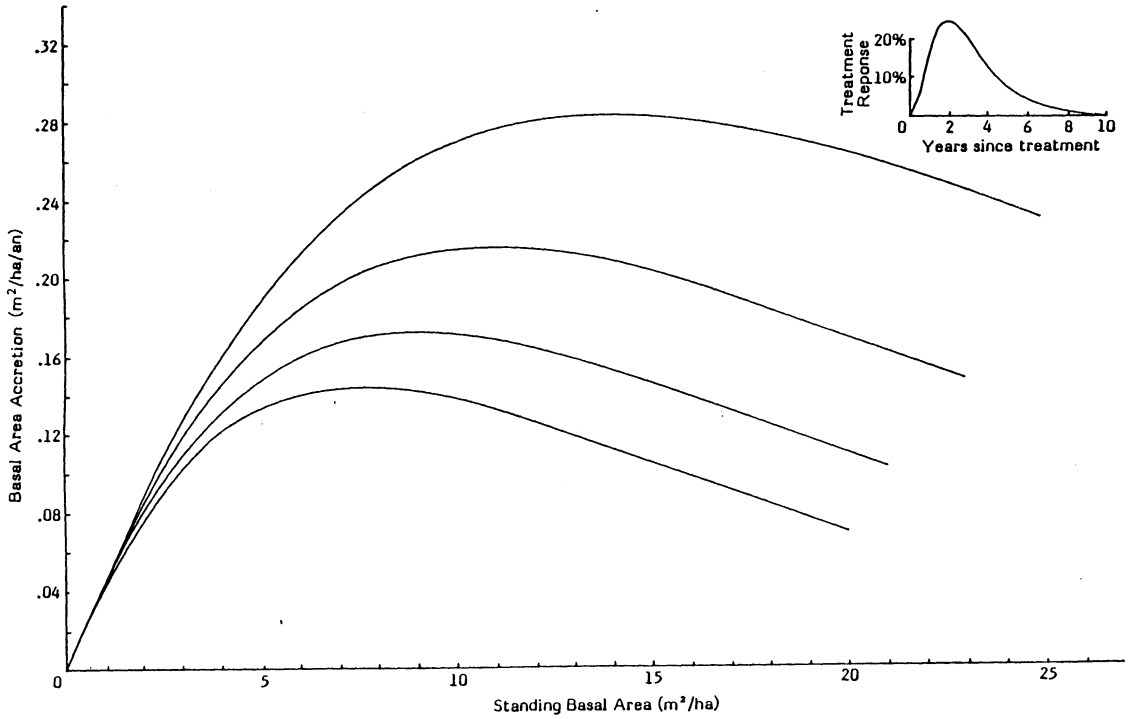

Figure 2 Stand Basal Area Accretion

Increment Distribution

Stand composition strongly influences the pattern of diameter increment distribution in cypress pine stands. Uniform cypress pine stands behave like exotic pine plantations in which tree basal area increments are proportional to tree basal areas (Haley 1955). In stands containing a wider range of stem sizes, the smaller stems get more than their "fair share" of increment, consistent with the model of Campbell et al. (1979) . In order to simulate this phenomenon, it is expedient to consider the cumulative proportion of basal area increment, and express it as a function of the cumulative proportion of basal area (Fig. 3):

$$Y = bX + (1-b)X^c$$

where

$$b = \exp(-\exp(0.8185-20.95/D_{max}))$$

$$c = 1 + 1.984\ SDev/D_m$$

and Y is the cumulative proportion ($0<Y<1$) of the basal area increment, X is the cumulative proportion ($0<X<1$) of the standing basal area (all stems arranged in descending order of dbh), D_m is the stand mean diameter, SDev is the standard deviation of diameters, and D_{max} is the greatest observed diameter.

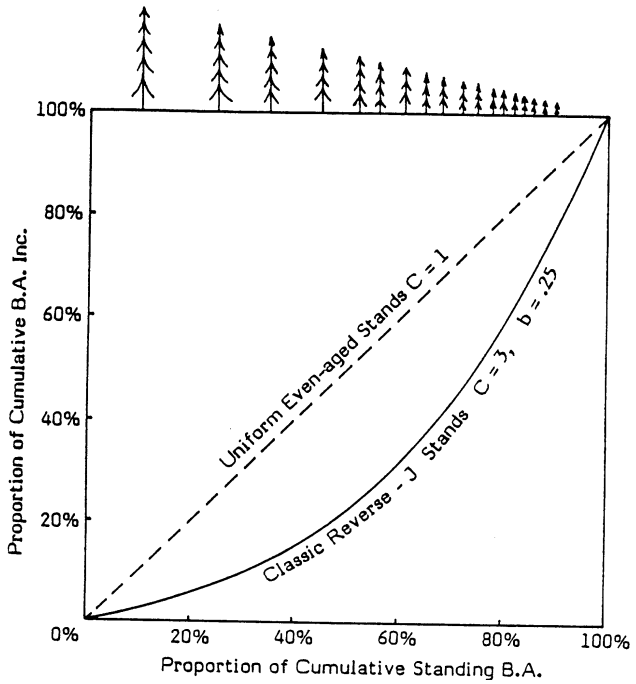

Figure 3 Increment Distribution

Individual Tree Increments

The resulting estimates of increment are checked to ensure that they do not exceed the potential diameter increment. In order to determine the maximum feasible individual tree increment, Newham (1964) assumed that the growth of open grown trees was the same as forest trees free of competition. Hahn and Leary (1979) contend that open grown trees may exhibit a different distribution of increment between bole and branches, and are thus unsuited for comparison with forest trees. Hahn and Leary (1979) used the 95th percentile of observed growth to approximate the competition free growth. Vanclay (1983, p.102) argues that this is likely to yield a biased estimate as it will favour increments attained during good seasons and determined from positively biased measurement errors. It is more appropriate to use increments from trees subjectively determined as free of competition.

Diameter increment is commonly predicted from empirical functions, often using simple quadratic relationships but also using complex equations derived from stepwise regression analysis. More robust equations proposed by Botkin et al. (1972) and Hahn and Leary (1979) offer some improvement, but may not adequately describe the data. Vanclay (1983, p.94) proposes the following equation, here derived from stems subjectively assessed to be free of competition (Fig. 4):

$$R = (-0.06357+0.007809SQ)*\exp(-0.08006BA)*\exp(0.3286TST^2\ e^{-TST})$$

$$DI = R * D((100/D)^{0.5258}-1) *\exp(-0.04421D)$$

where DI is the diameter increment ($cm.y^{-1}$). D is diameter (cm dbhob), SQ is site form (m), BA is stand basal area (m^2ha^{-1}), and TST is time since last silvicultural treatment (yr).

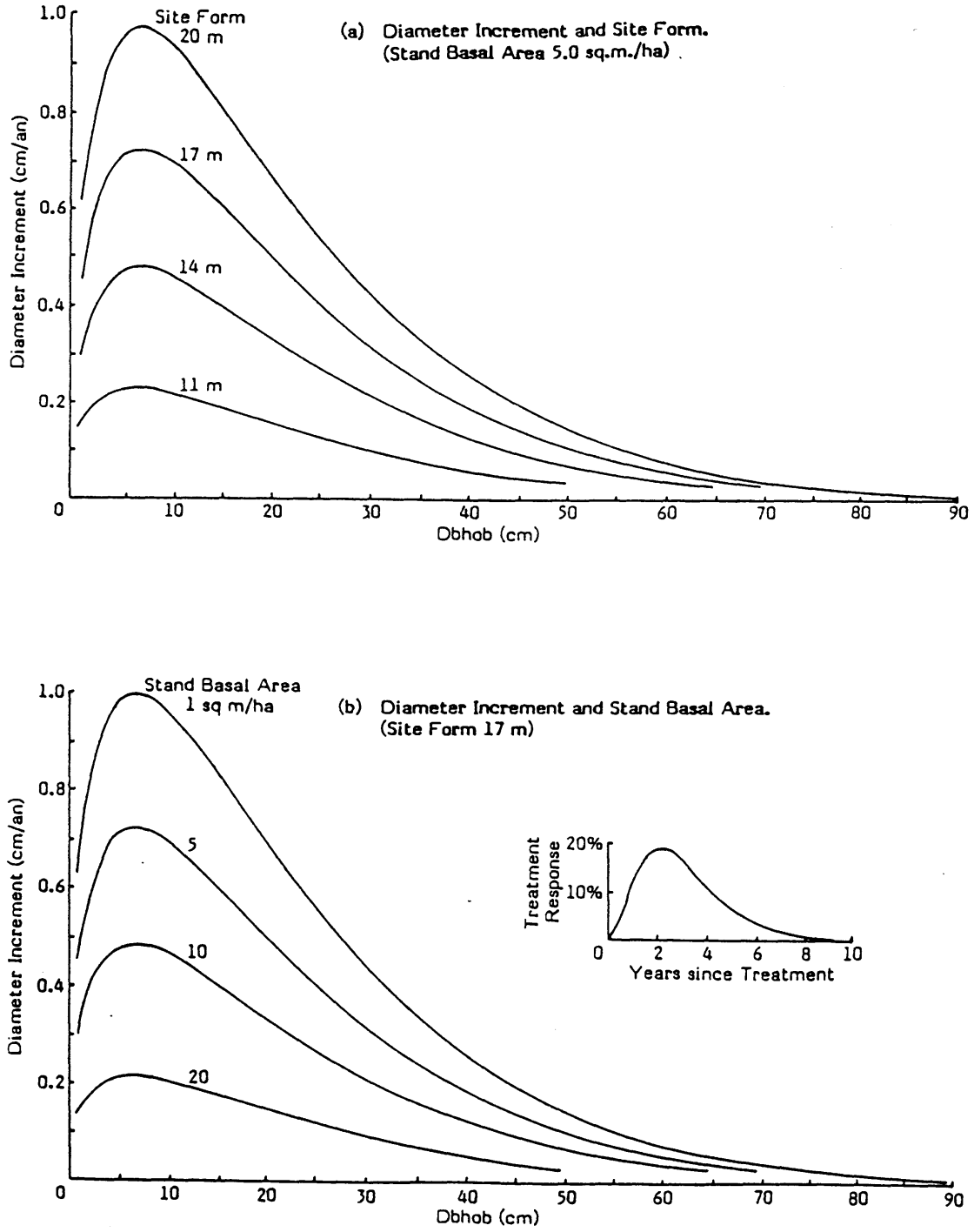

Figure 4 Potential Diameter Increment.

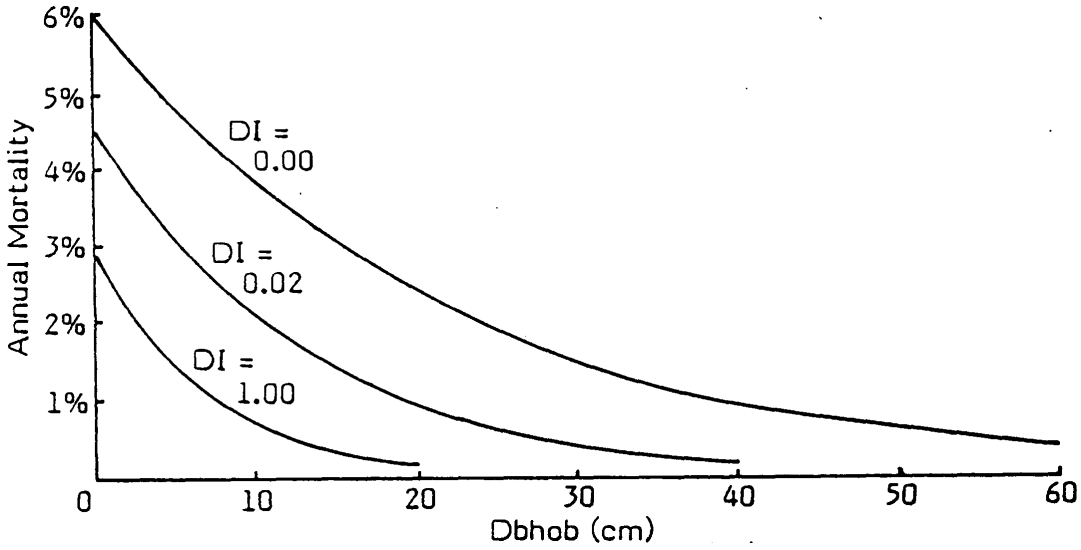

Figure 5 Mortality

Mortality

The Cypress Pine Growth Model addresses only regular natural mortality. Catastrophic mortality (wildfire, windstorm, etc) is ignored, and mortality arising directly from logging operations (smash, etc) is supplied by the user when logging procedures are nominated.

Mortality, like increment, may be modelled at the tree or stand level, and the stand level approach is generally more robust. Stand mortality may be predicted from stocking/density relationships such as the "self thinning rule" (Yoda et al. 1963). However, such approaches neither indicate which stems die, nor account for mortality not related to competition.

Prediction of mortality at the tree level appears more satisfactory. The model employs the logistic approach (Fig. 5) proposed by Hamilton (1974), *Buchman* (1979) and Ek t1980)':

R = 3.588+0.1433 D-0.02933/(DI+0.035)-0.003347 D/(DI+0.035)

M = 1/(l+exp(R))

where M is the proportion of stems dying in any year, D is dbhob and DI is the predicted diameter increment. It is important that predicted and not observed diameter increment is used in the model to avoid bias (e.g. drought may reduce increment and increase mortality). The inclusion of stand basal area in this relationship is currently being investigated.

Regeneration

In distance independent models, regeneration generally must be predicted at the stand level. Regeneration can normally be modelled at the tree level only when spatial information is available (e.g. assume a doughnut shaped occurrence of regeneration about a seed tree, the outer radius determined by seed dispersal and the inner radius determined by competition).

Cypress regeneration is modelled as cohorts of stems of similar heights. Regeneration reaching 0.1 metres height in any year is predicted as Figure 6:

$$R = \exp(0.4974\ SQ - 4.449 - 0.01723\ BASQ) - 1$$

and is assumed to grow at a rate given by Figure 7.

$$HI = 0.1673\ H - 0.2249 + 0.02794\ SQ - 0.1097\ \ln(BA+1)$$

(where HI is annual height increment, (m y^{-1}), SQ is site form, BA is stand basal area, H is tree height) until it reaches 3.0 metres when it is recruited to the main model at 2.0 cm dbhob.

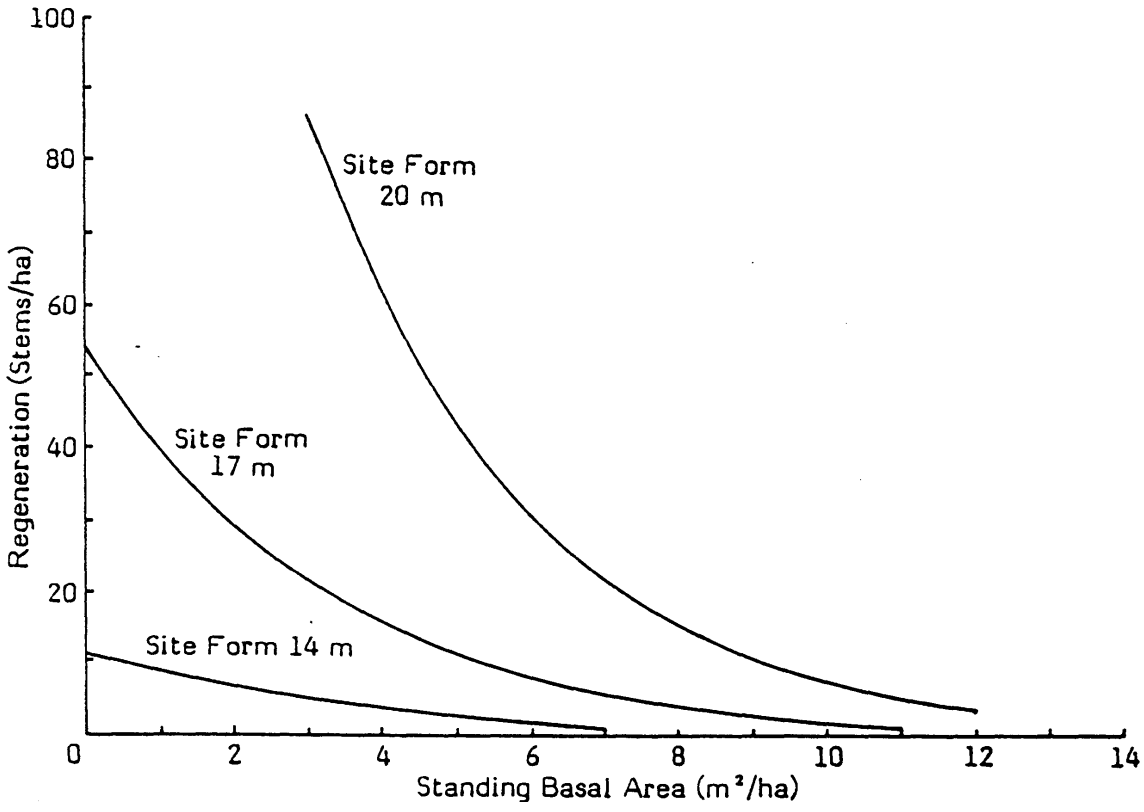

Figure 6 Regeneration

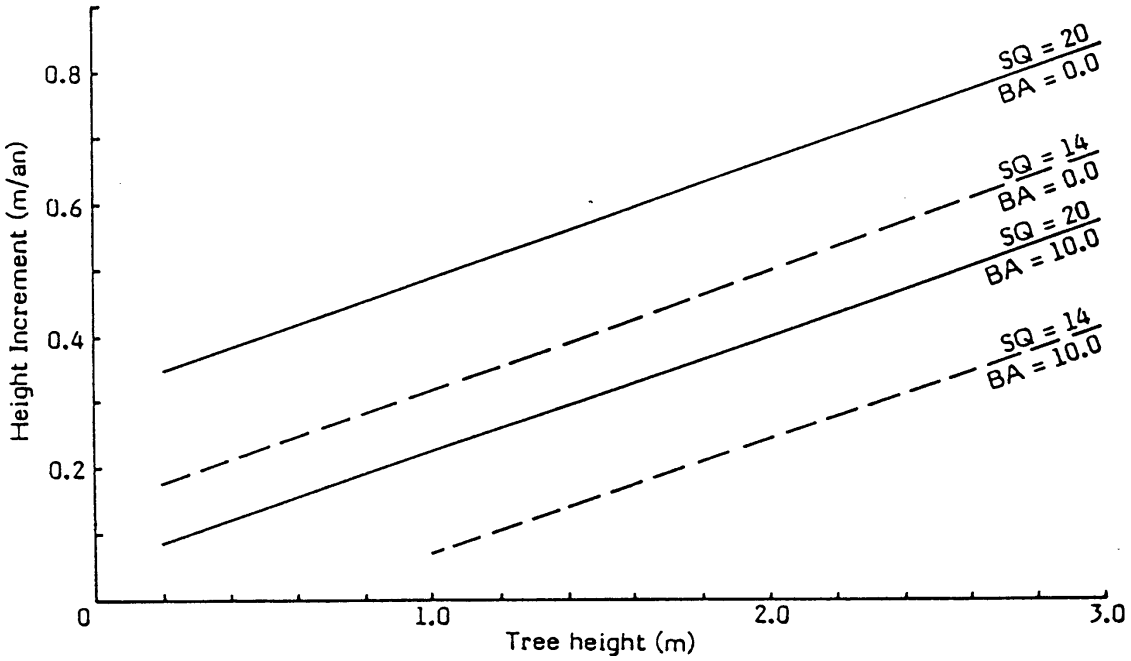

Figure 7 Height Increment of Regeneration

Validation

Validation is an important and integral part of growth model construction, and serves three main purposes. It should detect any errors which may exist in the model, should indicate the range of forest conditions over which the model may be expected to give satisfactory predictions, and should increase user confidence in the model. Leary et al. (1979) suggest three types of error which may affect growth models:

1. The component functional relationships may be inappropriate for describing the processes of growth;
2. The manner of combining these equations may be incorrect; and
3. The numerical constants obtained during development may not govern the system.

In order to detect any of these possible errors, the process of validation should comprise three stages (Vanclay 1983, p.129):

1. Each of the component functional relationships should be tested to ensure they are appropriate and accurate;
2. The computer program should be checked, and the modeller should ensure that the model behaves as expected and/or intended; and
3. Predictions by the model should be compared with permanent plot data not used in the development of the model and derived from a wide range of stand conditions.

As. the model has not yet been finalized, validation has not yet been attempted. Stand and tree increment functions have been validated using independent data and were shown to be unbiased and precise. Other functions are subject to further modification, and full validation will not be attempted until these functions have been finalized.

Discussion

Time since last silvicultural treatment is a variable in both stand and tree increment equations. Although it is preferable to employ only equations with variables derived from the current stand (site quality, basal area, dbhob, stocking, etc), these variables could not eliminate the strong pattern of residuals with time since treatment. This is consistent with observations by Johnston (1978) who could not identify the reasons underlying this response.

In modelling and yield regulation work it is necessary to quantify the site quality. In even-aged stands of known age, site index and similar measures are useful. However, in stands of mixed or unknown age, other measures are necessary. Site form, the expected height of a 25 cm dbhob tree is analogous to site index, and appears to provide a useful quantitative estimate of site productivity (Vanclay and Henry 19a8). Extensive testing reveals that site form is consistent with other concepts of site productivity including visual appearance (eg. Lewis et al. 1976, p.29), stand top height (Westveld 1933, Havel 1975), natural basal area (Havel 1980), and volume increment; is relatively unaffected by management (silvicultural treatment and logging); and is correlated with both diameter and stand basal increment.

The cohort approach has proved to be a particularly successful approach to modelling native forest stands. It also works well for stands with several species and is the basis for an interim growth model for rainforest in north Queensland (Vanclay- et al. 1985). The cohorts of the rainforest model identify the species group as well as stem size and stocking. The internal representation for each species group comprises three digits, one indicating the volume equation to be used, one indicating the logging rule to be applied, and one indicating the growth functions (diameter increment, mortality, recruitment) to be used.

One of the most interesting aspects of this model is the potential diameter increment function. The function is based on the Bertalanffy equation (von Bertalanffy 1951, Pienaar and Turnbull 1973), but contains a modification which ensures a better fit to the data, and accounts for the fact that a tree is not a solid mass of living tissue, but is a "thin layer of living sepulchre enclosing a growing corpse" (Harper 1977, p.218). In this modification, the term exp(-cD) represents the proportion of the tree volume which is living tissue:

$$DI = f(SQ) * \exp(-a\ BA) * D((Dmax/D)^{**}b - 1) * \exp(-c\ D)$$

This equation requires non-linear regression, and for all but the most comprehensive data sets, requires an independent estimate of Dmax, the maximum attainable diameter. However, an alternative equation of very much the same shape but with no theoretical basis, can be fitted by linear regression:

$$Ln(DI) = a + b\ Ln(SQ) + c\ Ln(D) - d\ D - e\ BA$$

This equation also avoids the need to specify a maximum attainable diameter.

The non-linear equation above may be further developed to include a term indicating the relative competitive status of the tree:

DI= f(SQ) * exp(-a BA) * D(g(CS)*(Dmax/D)**b - 1) * exp(-c D)

where g(CS) represents the competitive status of the subject **tree**. '

This theoretically represents a reduction in photosynthesis witn little change in respiration. The effect **is to** reduce the predicted increments and the maximum attainable diameter for any stem for which g(CS) is less than unity.

In distance dependent models, some measure such as Beauregard's (1976) crown view index or Solomon's (1980) relative crown area may be appropriate, but many other measures (see e.g. Vanclay 1983, p.102-111) may be suitable.

In distance independent models the relative competitive status of the tree may be computed as a generalization of Beer's Law (Shugart 1984, p.52) such as

$$g(CS) = \exp(-k*ST)$$

where ST is the sum of some attribute (height, diameter, etc) of the trees larger (taller, greater dbh, etc) than the subject tree.

A further possible development of this equation concerns the term exp(-c D) which represents the proportion of living tissue. This proportion is currently estimated as a simple function of stem size, but two possibilities offer interesting alternatives.

1. Suppression and slow growth rates may reduce the proportion of living tissue, and the term exp(-c D) could be replaced with a simple function of the recent periodic annual diameter increment and the current diameter such as Min[r*PDI/D, 1.0]. This could be useful in modelling growth of previously suppressed stems after release through logging or treatment.
2. As sapwood area exhibits a constant relationship with leaf area (Smith et al. 1966, Waring et al. 1982), this proportion could be expressed as a *function* of crown variables such as spread, depth and density.

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